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FUZZY MATHEMATICAL PROGRAMMING APPLIED TO THE LORIE-SAVAGE PROBLEM

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*Fuzzy Mathematica Programming Applied to the Lorie-Savage Problem.

Abstract.- The capital budgeting problem, formulated and partially solved by Lorie-Savage (1955), to choose projects when the capital budget is fixed, maximizing the net present value, is in this paper formulated applying fuzzy mathematical programming, expanding the approach used by H. Martin Weingartner (1967).

The Lorie-Savage Problem.

James H. Lorie and Leonard J. Savage in their article : "Three Problems in Rationing Capital", present the problem confronted by a firm which must ration available capital or liquid resources -- among competing investment opportunities. They discuss three problems in the rationing of capital, in the sense of liquid resources.

1.- Given the cost of capital, what group of investments should be selected ?

2.- Given a fixed sum for capital investment, what group of investment proposals should be undertaken ?

3.- How should a firm select the best among mutually exclusive alternatives ?

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A Mathematical Programming Approach to the Lorie-Savage Problem.

H. Martin Weingartner in his dissertation : "Mathematical Programming and the Analysis of Capital Budgeting Problems", emphasizes that the Lorie-Savage problem is concerned with a firm confronted with a variety of possible investment projects and a fixed capital budget, where the cash flows associated with each project are given and the firm's cost of capital is assumed to be known and to be independent of the investment decision, thus making the calculation of a net present value for each project possible. This net present value is defined as the algebraic sum of the elements of its stream of cash receipts and outlays discounted by the cost of capital. The objetive is then to select from among the projects with positive present values the particular projects which lead to the highest present value for the firm.

The foregoing formulation was expressed by H. M. Weingartner in the context of mathematical programming, specifically integer programming, in the following terms :

$$\text{Max} \quad Z = \sum_{j=1}^{n} b_j x_j$$

subject to :

$$\sum_{j=1}^{n} C_{tj} X_j \leq C_t, \quad t = 1,2,\ldots,T \rightarrow (1)$$

$$0 \leq X_j \leq 1$$

$$x_j \text{ integer}$$

where :

C_{tj} = cost of project j^{th} in the period t.

C_t = budget ceiling in the period t.

b_j = the present value of all revenues and costs associated with j^{th} project.

x_j = decision variable, which implies that a project j^{th} - is either accepted (i.e, $X_j^{\circ} = 1$) or it's rejected - (i.e. , $X_j^{\circ} = 0$).

Then (1) is a simple model for selecting among independent alternatives those projects where total present value is maximum, but where total outlay in each project falls within the budget limitation.

A Fuzzy Mathematical Approach to the Lorie-Savage Problem.

Let us consider the case in which what is looked for is not the extremization of Z but the achievement of a certain level of aspiration (yield). In the same way, let us suppose that each - of the restrictions can be violated up to a certain tolerance level according to membership functions associated with budget ceilings C_t ; and finallylet us add the restriction that no more than C dollars will be spent during the planning horizon of T periods.

The problem thus set forth can be formulated as :

$$\max \quad \tilde{Z} = \sum_{j=1}^{n} b_j x_j$$

subject to :

$$\sum_{j=1}^{n} C_{tj} X_j \underset{\sim}{\leqq} C_t \;, \; t = 1,2,\ldots,T$$

$$\sum_{t,j} C_{tj} x_j \leqq C$$

$$0 \leqq x_j \leqq 1$$

x_j integer .

Where the symbol ~ indicates that what is established is fuzzy .

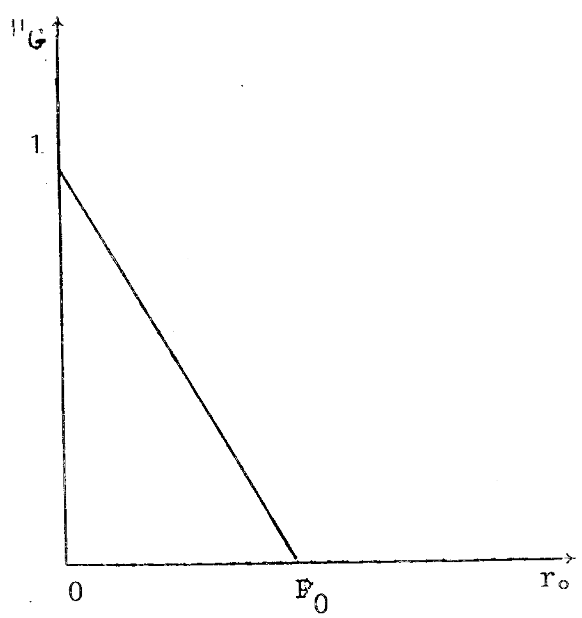
Let us suppose that the level of aspiration is about Mo dollars, associated with the membership function.

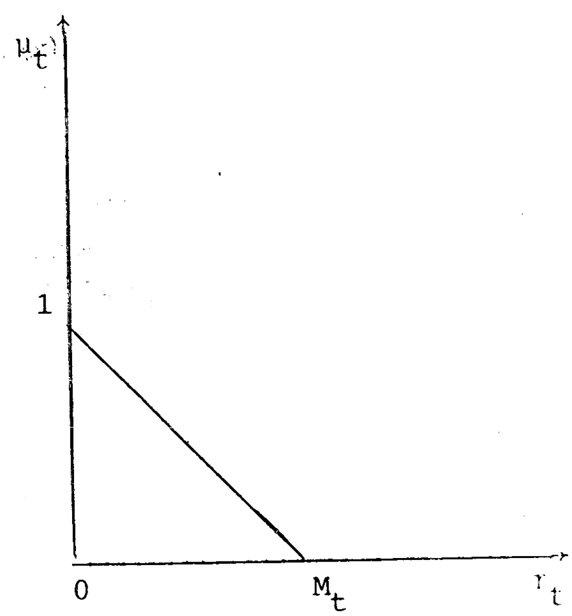
$$\mu_G = \begin{cases} 1 - \frac{r_o}{P_o} \;, & \sum_{j=1}^{n} b_j x_j = M_o - r_o \text{ and } r_o \geqq 0. \\ 1 \;, & \sum_{j=1}^{n} b_j x_j \geqq M_o \end{cases}$$

And let us suppose that the decision maker is predisposed to tolerate a violation of r_t (until $M_t > 0$), in the t restriction, according to the following membership function :

$$\mu_t = \begin{cases} 1 - \dfrac{r_t}{M_t}, & \sum_{j=1}^{n} C_{t_j} x_j = C_t + r_t,\ r_t \geq 0. \\ 1, & \sum_{j=1}^{n} C_{t_j} x_j \leq C_t \end{cases}$$

Graphically, these belonging functions are in the form :





Therefore, the foregoing problem may be expressed as :

$$\max \alpha$$

subject to :

$$P_0\alpha + r_0 \leqq P_0 \quad ; \quad r_0 \leqq P_0$$

$$M_t\,\alpha + r_t \leqq M_t \qquad t = 1,2,\ldots,T$$

$$r_t \leqq M_t \qquad t = 1,2,\ldots,T$$

$$-\sum_{j=1}^{n} b_j\, x_j - r_0 \leqq -M_0$$

$$\sum_{j=1}^{n} C_{tj}\, x_j - r_t \leqq C_t \;,\; t = 1,2,\ldots,T$$

$$\sum_{t,j} C_{tj}\, x_j \leqq C$$

$$0 \leqq x_j \leqq 1 \qquad j = 1,2,\ldots n$$

$$x_j \text{ integer}$$

$$\alpha \in R, \quad r_t \geqq 0, \qquad t = 1,2,\ldots,T$$

which corresponds to a problem of mixed linear programming, the solution of which be found by using conventional algorythms, where the optimal $(\alpha^\circ, x_1^\circ, \ldots, x_n^\circ,$

$r_0^o, \ldots, r_T^o$) means the following:

α^o = degree of membership of the optimal solution, taking into account membership functions of the objetive and restricciones, i.e., the total satisfaction of the problem or the restrictions and objetive function are satisfied in this order (where $0 \leq \alpha^o \leq 1$).

x_j^o = if it is 0, the j^{th} project is rejected; if it is 1, it is accepted.

r_t^o = In so far as the t^{th} restriction was violated, i.e., in so far as the available amount M_t was increased so as to be invested in the period t.

Moreover, it is easy to calculate $Z^o = \sum_{j=1}^{n} b_j x_j^o$.

Conclusions.- Because a fuzzy set is a generalization of the ordinary set, the ideas set forth by Lorie-Savage and formulated in the context of integer programming by H.M.Weingartner can be generalized in the context of fuzzy mathematical programming the decision maker will be benefitted because he will be able to incorporate his preferences, vaguerities and intuitions - in the model; this will result in a more appropriate solution than if the -- problem had been dealt with through conventional mathematics.

Bibliography:

1.- Weingartner, H.Martin (1967) "Mathematical programming and the analysis of capital budgeting problems". Markham Publishing Company. A Ford Foundation Prize Dissertation.

2.- Lorie, J.H., and Savage, L.J. (1955) "Three problems in rationing capital". Journal of Business.

3.- Hamacher, H., Leberling, H., and Zimmermann, Hans-J, -- (1978) "Sensitivity analysis in fuzzy linear progra -- mming". Fuzzy Sets and Systems 1, 269-281.

4.- Nahmias, Steven (1976). "Fuzzy variables", ORSA/TIMS.

5.- Negoita, C.V. and Saluria, M. (1976). "On fuzzy mathematical programming and tolerances in planning". Economic Computation and Economic Cybernetics Studies and Research, No.3.

6.- Ralescu, D.A., and Negoita, C.V (1974). "Aplications of fuzzy sets to systems analysis". Bucaresti-Piata, Scintelii, Editura Tehnica, NR.1.

7.- Tanaka H., Okunda, T. and Asai, K. (1974). "On fuzzy mathematical programming" J. of Cybernetics, 3,4, -- 37-46.

8.- Zadeh, Lofti A. and Bellman, Richard E. (1970). "Decision making in a fuzzy environment" Management --- Science, Vol.17, No.4.

9.- Zadeh, Lofti A. (1965) "Fuzzy Sets". Information and Control, Vol. 8, 338-353.

10.- Zimmermann, Hans-J. (1976). "Description and optimization of fuzzy sets". Int. J. General Systems, Vol. 209-215.

11.- Zimmermann, Hans-J. (1978). "Fuzzy programming and linear programming with several objetive functions". Fuzzy Sets and Systems I, 45-55.

12.- Zimmermann, Hans-J (1979). "Theory and applications of fuzzy sets". k.b. Haley, Ed., OR178.1017 , 1033.